Problem:-1 :- Three masses m_1, m_2 and m_3 are of respectively 3kg, 4kg and 2kg and rotating at radii of 30mm, 20mm and 25mm respectively. The position of masses m_1, m_2 and m_3 with horizontal axis is at an angle of 30°, 120° and 270° respectively. Find the balancing mass attached at the radius of 35 mm from axis and its position with horizontal.

⇒ **Solution :-**

→ $m_1 = 3$ kg $r_1 = 30$mm $\theta_1 = 30°$

→ $m_2 = 4$ kg $r_2 = 20$ mm $\theta_2 = 120°$

→ $m_3 = 2$ kg $r_3 = 25$ mm $\theta_3 = 270°$

→ $m = ?$ $r = 35$mm $\theta = ?$

⇒ Resolving m_1r_1, m_2r_2 and m_3r_3 in horizontal direction

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3$$

$$= 3 \times 0.03 \times \cos 30° + 4 \times 0.02 \cos 120° + 2 \times 0.025 \times \cos 270°$$

$$= 0.078 + (-0.04) + 0$$

$$\therefore \Sigma H = 0.038 \text{ kg.m}$$

⇒ Similarly resolving in vertical direction

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$$

$$= 3 \times 0.03 \times \cos 30° + 4 \times 0.02 \times \cos 120° + 2 \times 0.025 \times \cos 270°$$

$$= 0.045 + 0.069 + (-0.05)$$

$$\Sigma V = 0.064 \text{ kg.m}$$

→ Now, Resultant centrifugal force

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(0.038)^2 + (0.064)^2}$$

$$\therefore R = 0.0744 \text{ kg.m}$$

⇒ Now, $r = 0.035$ m

$$\therefore R = m \cdot r$$

$$\therefore m = \frac{R}{r} = \frac{0.0744}{0.035}$$

$$\therefore m = 2.126 \text{ kg}.$$

∴ Balancing mass,

$$\boxed{m = 2.126 \text{ kg}}$$

→ Now, $\tan\theta' = \frac{\Sigma V}{\Sigma H} = \frac{0.064}{0.038}$

$$\therefore \tan\theta' = 1.6842$$

$$\therefore \theta' = \tan^{-1}(1.6842)$$

$$\theta' = 59.3°$$

⇒ Now Angle of Balancing mass,

$$\theta = 180° + \theta'$$

$$= 180° + 59.3°$$

$$\boxed{\theta = 239.3°}$$

Problem:-2:- Two masses of 8 kg and 16 kg rotates in the same plane at radii of 1.5 m and 2.25 m respectively. The radii of these masses are 60° apart. Find the position of the 3rd weight of the magnitude of 12 kg in the same plane which produce complete dynamic balance of the system.

⇒ **Solution :-**

→ $m_1 = 8$ kg $\quad r_1 = 1.5$ m

→ $m_2 = 16$ kg $\quad r_2 = 2.25$ m

→ $\theta_1 = 0°$ $\quad \theta_2 = 60°$

→ $m = 12$ kg $\quad r = ?$ $\quad \theta = ?$

⇒ Resolving masses in horizontal direction

$\therefore \Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2$

$= 8 \times 1.5 \times \cos 0° + 16 \times 2.25 \times \cos 60°$

$= 12 + 18$

$\Sigma H = 30$ kg.m

⇒ Resolving masses in vertical direction

$\therefore \Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2$

$\therefore = 8 \times 1.5 \times \sin 0° + 16 \times 2.25 \times \sin 60°$

$= 0 + 31.18$

$\therefore \Sigma V = 31.18$ kg.m

⇒ Resultant force

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$\therefore R = \sqrt{30^2 + 31.18^2}$

$\therefore R = 43.27$ kg.m

→ Now,

$R = m \times r$

$\therefore r = \frac{R}{m} = \frac{43.27}{12}$

$r = 3.6$ m

$\therefore$ Radius of Balancing mass

$\boxed{r = 3.6\text{ m}}$

⇒ Now

$\tan\theta' = \frac{\Sigma V}{\Sigma H} = \frac{31.18}{30}$

$\therefore \tan\theta' = 1.039$

$\therefore \theta' = \tan^{-1}(1.039)$

$\therefore \theta' = 46.1°$

→ Angle of Balancing mass

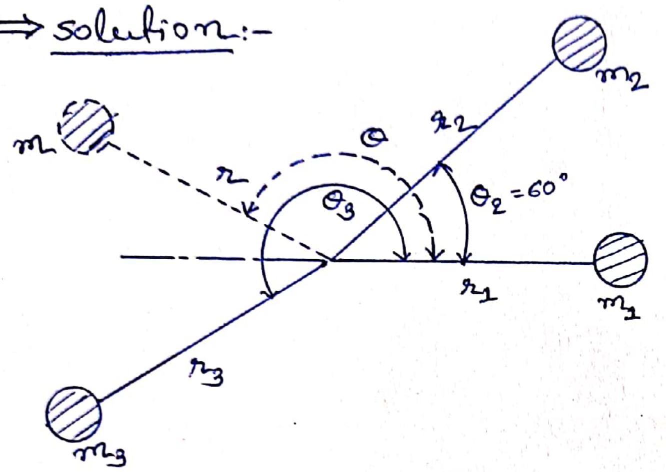
$\theta = 180° + \theta'$

$= 180° + 46.1°$

$\boxed{\theta = 226.1°}$

Problem :- 3 :- Three masses 5 kg, 6 kg and 8 kg are revolving about an axis in the same plane at the radii of 0.12 m, 0.10 m, and 0.15 m respectively. The angle betn 5 kg and 6 kg mass is 60° and 6 kg and 8 kg mass is 165°. Determine magnitude and position of the balancing mass at the radius of 0.14 m for the state of balance

⇒ solution:-



→ $m_1 = 5$ kg $r_1 = 0.12$ m $\theta_1 = 0°$ (Assumed)
→ $m_2 = 6$ kg $r_2 = 0.10$ m
→ $m_3 = 8$ kg $r_3 = 0.15$ m $\theta_2 = 60°$
→ $m = ?$ $r = 0.14$ m $\theta_3 = 60° + 165° = 225°$

⇒ Resolving masses in Horizontal direction

$\therefore \Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3$

$= 5 \times 0.12 \times \cos 0° + 6 \times 0.10 \times \cos 60° + 8 \times 0.15 \times \cos 225°$

$= 0.6 + 0.3 + (-0.849)$

$\therefore \Sigma H = 0.0515$ kg·m

⇒ Similarly,

$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$

$= 5 \times 0.12 \times \sin 0° + 6 \times 0.10 \times \sin 60° + 8 \times 0.15 \times \sin 225°$

$= 0 + 0.52 - 0.849$

$\therefore \Sigma V = -0.329$ kg·m

⇒ Resultant force,

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$\therefore R = \sqrt{(0.0515)^2 + (-0.329)^2}$

$\therefore R = 0.3329$ kg·m

→ Now, $R = m \cdot r$

$\therefore$ Balancing mass

$m = \frac{R}{r} = \frac{0.3329}{0.14}$

$\therefore \boxed{m = 2.38 \text{ kg}}$

⇒ Now,

$\tan\theta' = \frac{\Sigma V}{\Sigma H} = \frac{-0.329}{0.0515}$

$\therefore \tan\theta' = -6.39$

$\therefore \theta' = \tan^{-1}(-6.39)$

$\therefore \theta' = -81.1°$

⇒ Angle of balancing weight

$\theta = 180° + \theta'$

$= 180° - 81.1°$

$\boxed{\theta = 98.9°}$